MATHS POWERS

The following is based on the work of Professor John Mason (2005) who recognised that human beings possess natural abilities, which he terms “natural powers” which can significantly support and develop learning in mathematics. Throughout the Mathematics Specialist Programme you will be asked to become more aware of these powers both in your own learning and in that of the children and adults you work with.

Children have a natural desire to connect ideas and make sense of the world: Harry (aged 4) was asked “What is half of 12”, he looked perplexed and said if I cut it down the middle (dividing the digits) it is either one or two”. He had made a connection to cutting a shape in half; however, after been shown how to divide 12 objects into two equal groups of 6; he understood and declared “oh it is the opposite of doubling” He had connected the mathematics and put it into a context that he understood.

Mason identifies four sets of mathematical powers which support children in making sense of the world and learning mathematics. He makes reference to these powers in much of his writing.

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The following extract is taken from: Mason J, Graham A and Johnston Wilder S (2005) Developing Thinking in Algebra, London: Paul Chapman Publishing, Chapter 14.

Imagining and Expressing

Every child can imagine, whether they use mental pictures, words, kinaesthetic responses, have a vague sort of ‘sense-of ’, or some combination of these. It is by means of imagination that people are able to contemplate what is not actually present, whether it is a pattern continuing into the future or an experience from our past. Imagine standing at the entrance to your kitchen; which way would you turn to get a fork? These sorts of questions can be answered because it is possible to ‘be’, mentally, at the entrance of your kitchen. Sometimes your body knows the answer, so you imagine yourself moving and then watch which way you moved in order to find out. What can be expressed, whether in pictures, movement, words or symbols, is only a small part of that rich world called experience. Learning to write stories exercises connections between speech and imagery; learning to write algebraic symbols exercises connections between awareness of patterns and generality, and symbols. The advantage of algebraic symbols over words and pictures is that the symbols are more easily manipulated, once they have become familiar and confidence-inspiring...........

The power to imagine is usefully called upon explicitly and can be developed with practice. If imagination is not called upon in mathematics, then a powerful link to the emotions is neglected, and motivation-interest may suffer. If expression in multiple forms is not encouraged, then learners may form the mistaken impression that mathematics does not offer opportunities for creativity. If learners encounter a very limited range of images, and a very limited range of expressions, they are likely to form the erroneous impression that mathematics is a very limited domain of human experience.

Experiencing and expressing generality is entirely natural for learners. The issue is whether that power is being called upon and developed in mathematics lessons, or whether it is being left to one side where it is likely to atrophy, at least in the mathematics context, through non-use.

Specialising and Generalising

Very young children display the power to generalise as well as specialise: in order to talk, young children need to generalise, for words such as dog are by their very nature general, and they need to specialise when identifying a specific object described in words. Freudenthal (1979, p. 235) illustrates this beautifully in describing an incident with his grandson: Bastiaan starts to ask about two women, one in a wheelchair: ‘What did the lady say to the lady [hesitation] that pushed the wheelchair?’ He recognises in the moment the ambiguity and acts to correct this. Such ‘shifts’ are an integral part of learning to use language. If every experience was individual and unique, our brains would soon reach memory overload. It is precisely because people can generalise that they can categorise and so make sense of experience through stressing (and consequently ignoring).

The whole point of arithmetic is not to learn number facts, but to learn general methods. No one would imagine asking learners to memorise all possible two-digit additions. Rather, learners are expected to discern and integrate into their functioning, methods for doing such operations. Any method of doing an action in different situations or with different data is a generality. It involves the learner constructing if not expressing awareness of a generalisation. Indeed, even number names are generalisations, nouns abstracted from multiple contexts, so that 1, 2, 3 become as concrete and specific as 'one ball' , 'two balloons' , 'three bicycles' are for younger children...........

Generalising happens spontaneously when someone imagines a process continuing. It might be something simple like adding one, over and over, leading to the generality that there can be no largest integer because 'you can always add one more'. It might be something more complicated like imagining doubling and subtracting 1 over and over again, or thinking of a graph as extending on and on for ever in both directions, not just confined to what can be seen on a screen (mental or material). It might be something quite subtle such as that no matter what two numbers you add (or multiply) the answer is independent of the order, or that you can factorise any whole number uniquely into a product of prime numbers. Sometimes generalising is so quick that you do not realise there may be other ways of perceiving and hence generalising.

When you see the sequence 1, 2, 3, 4, 5 it is natural to assume the continuation 6, 7, 8, … . However, depending upon the context, the sequence may continue in a different way. It might be part of a bus list, in which the next term is 35.This is why it is vital, whenever a pattern is detected, whenever relationships are surmised, to base all conjectures on some agreed source which generates the sequence or pattern.

Hand in hand with generalising goes its reverse process, particularising, or as the famous Polish-American mathematician and mathematics teacher George Polya referred to it, specialising. Both processes are important, as Polya pointed out: 'we need to adopt the inductive attitude [which] requires a ready ascent from observations to generalizations, and a ready descent from the highest generalizations to the most concrete observations' (Polya, 1957, p. 7, 1962). The reason for trying a particular case of something more general is to try to see what is going on through the example, through watching how you do the example. The purpose of the specialising is to make sense, to enable a reconstruction of the general, expressed in a more familiar language and a more manipulable symbolism. For example, when faced with a conjecture such as 'the sum of an even number of consecutive odd numbers is divisible by double the number of numbers', trying some cases gives a sense of what the conjecture is about.

Conjecturing and Convincing

Conjecturing is a way of working, an ethos, in which ideas are developed through learners thinking out loud or explicitly in some other way. Everything that is said is thought about and tested by those who are listening. People speak because they are uncertain and hope to get some help from others in how to articulate what they think they are 'seeing' or thinking. It is a way of working in which everyone takes responsibility for making sense of what is said, and anybody can be asked to explain their thinking, that is, to try to convince others. One of the most important things that a school can contribute in the way of developing learners' powers is to engender a conjecturing atmosphere.

In a conjecturing atmosphere, when someone says something that is not quite understood by another, someone might ask for or offer an example, or might focus on a detail and ask pointedly for elaboration. In a conjecturing atmosphere, people do not say 'That's wrong', they say 'I invite you to modify your conjecture', or they say 'What about' and offer a possible counter-example. Mathematical thinking really only gets going when there are competing conjectures or when there is something to justify. If learners feel that answers are always either right or wrong, they may become reticent about offering their ideas. If conjecturing is valued, and especially if modifying conjectures is valued and praised, then mathematical thinking is more likely to flourish.

Conjecturing is about being aware of the status of some assertion: is it reasonable? Is it always true? Is it sometimes true? Is it never true? How do I know? If I cannot justify it by convincing someone else, then it remains a conjecture, something which I think may be true, and for which I have some possible evidence. Trying to justify why you think something is the case by trying to convince someone else is most helpful in sorting out what you think, for as you start to explain it, things either tend to fall into place or fall apart. The person being explained to can learn from the experience by learning to ask probing questions ('give me an example', 'what if … was different?', 'how do you know?', 'why must …?' and so on), which in turn develops expertise in convincing others. Thus learning to be sceptical when listening to others trying to convince is an important part of learning to convince people yourself, for you learn to internalise the sorts of objections that others are likely to make.

When ideas are coming thick and fast it is sometimes hard to hold on to what you think is actually the case. Consequently one of the features of a conjecturing atmosphere is making a record of current conjectures. Like all mathematicians, learners sometimes run out of time when exploring some idea. A sensible place to leave off work is to make a record of current conjectures, and a summary of available evidence. Developing this practice provides a satisfactory way of leaving a topic or a project and going on to something else.

Organising and Classifying

Human beings make sense by organising and classifying experience. Putting dishes and laundry away are forms of imposing order on the material world, and act as useful metaphors for working in the mental, symbolic worlds, and social worlds. One of the reasons for organising is that it reduces confusion. It simplifies the multitude of experiences and forces acting upon you. The desire to impose order is manifested very early. Every experience is classified (unconsciously) in order to assimilate it into current schema and so 'make sense of it'. If it resists classification, then it is either rejected out of hand or schemas are altered in order to accommodate it (Piaget, 1971). Having acknowledged the desire and value of organising, it is useful to probe just how it is that organising comes about. Each act of sorting involves stressing some (relevant) features and ignoring others, which in turn requires being able to discriminate those features. Sorting tasks are really excellent for getting groups of learners to express their thinking to each other and to negotiate different ways of seeing. Some learners will learn from others ways of discerning that had not previously come to mind. Others will find their way of perceiving supported or confirmed....